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ABSTRACT

Concepts important to cognitive development in children can be classified according to several levels. At level 1, concepts are equivalent in structural complexity to binary relations and univariate functions. At level 2, concepts are equivalent to compositions of binary relations, binary operations, and bivariate functions. At level 3, concepts are equivalent to compositions of binary operations. Research has shown that level 1 concepts can be attained by 2-year-old children, level 2 concepts by 5-year-olds, and level 3 concepts by 11-year-olds. This document presents and explains the evidence on which this conclusion is based, using transitivity, classification, and mathematical reasoning tasks for illustration. The practical utility of being able to define the limits to children's understanding in a general sense is discussed. (Author/RH)

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# How do we define the limits to children's understanding?[1]

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Abstract

The concepts which are important to cognitive development in children can be classified into several levels. At level 1 there are concepts which are equivalent in structural complexity to binary relations and univariate functions. At level 2 there are concepts which are equivalent to compositions of binary relations, binary operations and bivariate functions. At level 3 there are concepts which are equivalent to compositions of binary operations. Research has shown that level 1 concepts can be attained by 2 year old children, level 2 concepts by 5 year olds and level 3 by 11 year olds. The evidence on which this conclusion is based is presented and explained, using transitivity, classification and mathematical reasoning tasks for illustration. The practical utility of being able to define the limits to children's understanding in a general sense is discussed.

How do we define the limits to children's understanding?

One practical benefit from research on cognitive development should be to define just what children can and cannot do as a function of age. This has obvious implications for education, because it should permit curricula to be designed so that they conform to, and take maximum advantage of, children's capabilities. So far, however, we have seen little by way of satisfactory attempts to achieve this aim.

The most significant attempt so far has been the stage theory of Jean Piaget. According to this formulation there are four stages of cognitive development. Infants or children up to two years of age are in the sensorimotor stage, and are limited to presymbolic behaviour. From two to five years children are in the preoperational stage, which means they have a rich repertoire of symbolic processes but their thought operations are primitive and fraught with error. The six-to-ten year old child is in the concrete operations stage and can reason logically, but his/her reasoning is restricted to operations which reflect past experience, and it is not until the formal operations period, beginning in adolescence, that reasoning by hypothesis occurs.

In my view Piaget has drawn our attention to a multitude of striking and important phenomena in cognitive development and his original contributions to this area were far ahead of those of his competitors in the first half of this century. However, numerous difficulties have been found with Piaget's formulation (Brainerd, 1978) and other approaches to cognitive psychology have since overtaken it, so that it can no longer be regarded seriously as explicit theory.

One alternative to Piagetian theory is the experimental child psychology approach, which has challenged many of Piaget's main tenets. On the other

hand it has not produced any alternative way of defining children's capabilities, and has mostly contented itself with denying the validity of the limits implied by Piaget's stage theory. The other major alternative is the information processing approach, of which there are two kinds. The artificial intelligence approach, which is represented by the work of Klahr and Wallace (1976) and by the papers of Ian Wallace and Robert Neches in this symposium, has been very successful in creating models of real-time problem solving processes. It has not so far been concerned, however, to define children's cognitive capacities and limitations. The second kind of information processing approach is represented by the work of Pascual-Leone (1970) and Case (1974) who have generated predictions about children's capabilities based on the concept of a limited capacity central computing space. This model is closest to the one I want to present, which also proposes that children's capabilities can be predicted from information processing limitations. Although the two approaches are similar in this general sense, they differ in almost every other respect, as has been explained elsewhere (Halford, 1982; Halford & Wilson, 1980).

The current approach is based on a theory of representations. The concept of representation subsumes the concept of understanding, because to understand a problem is to be able to apply a representation to it which matches the structure of the problem. The limitations to children's thinking and understanding arise from two things. Firstly, more complex concepts require more complex representations. Secondly, more complex representations require more space in working memory. It follows then that the concepts which children can understand, and the thinking they can perform, are limited by working memory.

It is possible to sort concepts into three levels. At level 1 we have concepts which are equivalent to binary relations and univariate functions. Examples of concepts which belong to this level are size relations, such as one object being larger than another, or one set being larger than another (five apples is more than four apples). Less obvious examples are classifications by a single criterion. If we classify things by shape, then objects are placed together because they have the "same shape", which is a binary relation. Other concepts which belong to this level are those which involve functional relations. For example, the wind makes the trees move, or dropping a cup makes it break. This is one type of univariate function. But unidimensional patterns may also be expressed as univariate functions, so they also belong to this level. Research in our laboratories (Halford, 1980a,b; Halford & MacDonald, 1977) as well as by other people (e.g., Bullock & Gelman, 1977) has shown that children master concepts at this level between the age of two and five.

At level 2 we have concepts which are equivalent to compositions of binary relations, to binary operations and bivariate function. Concepts at this level have an extra dimension of complexity over those at level 1. Transitivity would be a good example. Transitivity is a binary operation, and as such is a composition of binary relations. A common type of transitive inference task involves presenting two relations and asking participants to infer the third: $a > b$, $b > c$, therefore $a > c$. The relations $a > b$ and $b > c$ are being integrated or composed into the ordered triple, $a,b,c$ which permits the inference $a > c$. There are two relations here as against one at level 1.

Our research has led us to the conclusion that children do not understand transitivity or other level 2 concepts until they are about five years of age (Halford, 1982; Halford, Note 1; Halford & Kelly, Note 2).

This brings us into conflict with well-known research by Bryant and Trabasso (1971) and a number of other researchers who followed them (see reviews by Thayer & Collyer, 1978; Trabasso, 1977; Halford, 1982, chapter 6, for details). Because of this conflict, and because the transitivity concept is very useful for illustrative purposes, we will consider this issue in detail.

One of our experiments involves presenting children with a set of five relations: a > b, b > c, c > d, b > d, d > e. We then ask them to place the middle three elements, b,c,d in order and we provide them with a memory aid to ensure they remember the relations presented. To make it easier still for them we let them perform the task in two steps; they first place bd, then add c. Both 3-4 year olds and 5-7 year olds perform the first step easily, because they can place bd in order by simply retrieving the relation b > d from the memory aid, or from their own memories. When they have to place c however, the 3-4 year olds are in serious trouble. This step is harder because two relations have to be considered; i.e. we must recognize that b > c and c > d in order to place c correctly between b and d. Consideration of only one relation yields ambiguous orders. For example, if we consider only b > c, we obtain the order bcd or bdc. Similarly, if we consider c > d, we obtain the orders bcd or cbd. It is only by considering both that b > c and c > d that we unambiguously recognize that bcd is the correct order.

In another experiment we present children with the relations a > b, b > c, c > d, d > e, then ask them to place bd in order. This is formally equivalent to a transitive inference task, but it also requires two relations to be considered: we must recognize that the correct order is bd because b > c and c > d. Again 3-4 year olds had difficulty, but children over 5 succeeded quite easily. This is despite the fact that the 3-4 year olds readily placed bc or cd in order. They could consider any one of the

relevant relations, but could not integrate two relations in a single decision (Halford, Note 1).

Why have preschool children failed our transitive inference task when they apparently passed the equivalent task presented by Bryant and Trabasso (1971) and others who followed their procedure? There are several reasons for not regarding the Bryant and Trabasso paradigm as a valid test of transitivity, and these are spelled out in detail elsewhere (Halford, 1982, chapter 6; Note 1), but in essence they amount to the fact that genuine transitive inference and ordering tasks entail integrating two relations, but the Bryant and Trabasso procedure at no stage requires that performance. Our data shows furthermore that it is beyond the abilities of preschool children.

We have also shown that the process of integrating two relations in transitive inference tasks imposes a load on working memory beyond that imposed by considering any one relation (Halford, Bain & Maybery, in press). The inability of preschool children to integrate premises may well be due therefore to lack of working memory capacity. In general we have found that their capacity is restricted in the sense that they can consider only one relation at a time, and cannot deal with concepts which entail integrating two relations into a single representation or a single decision.

In classification tasks we have again found that preschool children's working memories seemed to be insufficient to permit them to consider more than one attribute, with the result that they were once again restricted to level 1. The task is illustrated in Figure 1. Participants were shown a matrix with one row and one column filled, and were asked to select an element from one of three alternatives to fill a specified cell. At level 1, the three alternatives were always correct in one of the relevant dimensions, and the choice had to be made on the basis of one dimension only. For

instance, in the first case shown all the choice elements are crosses, so shape can be ignored and a choice made from the three colours. At level 2 however both colour and shape had to be considered in order to select the correct element.

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Insert Figure 1 about here

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Children were given learning set training with a series of such tasks. The acquisition criterion for such a learning set is ability to correctly choose elements of a new problem. That is, the terminal point of learning set training corresponds to correct performance on the matrix classification task. All of the children with mental ages over five years reached criterion on both level 1 and level 2 tasks. Among children with MA less than five, all were successful on level 1, but only 20 percent succeeded on the level 2 task, and examination of the learning curves of the others showed no tendency to improve.

Notice that the two tasks differ only in the amount of information which has to be utilized in making a correct choice. The level 1 tasks required colour or shape information to be used equally often, but the two kinds of information never had to be integrated into a single decision, whereas at level 2 they did. It seems a reasonable conclusion, therefore, that children below MA five years were unable to process enough information to perform the level 2 task.

At level 3 we have concepts which are equivalent to compositions of binary operations, or trivariate functions. The difference is illustrated by the two types of item in Table 1. At level 2 we have a problem in which one operation, 3 x 5, is specified, but the second operation is not. To solve the problem it is necessary to recognise that the unknown second operation is subtraction--thus (3 x 5) - 5 = 10. In the second type of

item there are two unknown operations, and it must be realised that the solution to the problem is (4 + 2) ÷ 3 = 2. The first problem is level 2 because the decision to be made can be represented by a single binary operation, whereas the decision in the second problem must be represented by a composition of two binary operations. This implies that we cannot know we are right about one operation without knowing we are right about both of them. Thus, just as decisions about level 2 concepts entailed considering more information in a single decision than decisions about level 1 concepts, decisions about level 3 concepts entail more information than decisions about level 2 concepts. In the present example we must decide about both operations together at level 3, but only about one operation at level 2.

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Insert Table 1 about here

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Our research generally supports the proposition that children cannot master level 3 concepts until the age of 11 years on average. This is illustrated by the data in Table 1, where the proportion of children who succeed are shown as a function of grade. It is not until grade 6, when the children were approximately 11 years mental age, that the 50 percent success rate was achieved on the level 3 task. We have also examined the working memory loads imposed by these two levels of task, using the concurrent memory task technique described earlier for transitive inference. Again it was shown that the level 3 task imposed a higher load on working memory (Halford, Bain & Maybery, Note 2). It seems, therefore, that children cannot integrate enough information in working memory for level 3 tasks until they are about 11 years of age. Note however that they could still solve a problem like X = (4 + 2) ÷ 3, or even more complex versions of such tasks, with more steps and larger numbers. Why can they do this

but not perform the apparently equally simple level 3 task in Table 1? The reason is the amount of information which must enter into each decision. With the equation above, we can close or perform the first operation alone, without attending to the second operation at all. Thus working memory is only occupied with one operation at a time, but does not need to encompass information about two operations in a single decision. Such a task is, therefore, level 2.

We have also shown that children do not seem to be able to encode three-dimensional patterns reliably until 11 years (Halford, 1980b). Theoretical predictions have been made that children will not be able to test hypotheses in rule identification tasks which involve two-dimensional concepts of the type defined by Haygood and Bourne (1965), nor will they be able to test hypotheses in attribute identification tasks involving conditional and biconditional concepts until 11 years of age. These predictions have been spelled out in more detail elsewhere (Halford, 1982, chapter 9).

Practical implications

The foregoing research appears to have demonstrated that there are clear limits to the concepts which children at a particular age can master. These limits are defined by the amount of information which must be integrated into a single decision or a single representation. Thus preschool children seem unable to master transitivity, the concept of serial order, or multiple attribute classification, because they cannot integrate information about two binary relations in a single decision or representation. Similarly children below the age of 11 years on average cannot integrate enough information in working memory to deal with concepts which are defined by a composition of two binary operations.

We still have a lot to learn about the nature of working memory and its development, and current work in our laboratory is also aimed at making a contribution to that problem for which there is already a burgeoning literature (see Dempster, 1981, or Hitch, 1980, for reviews). While I do not want to beg any of the complex and important questions about the nature of working memory and its development, I nevertheless think it is fair to say that certain limitations have been demonstrated in the kinds of reasoning and decision making which children can perform. These limitations are clearly related in some way to the amount of information which must be integrated in a single decision, so there is a sense in which working memory must be involved.

We can, therefore, define the limitations to children's understanding and reasoning in a sense which is more general than any one task or even any one type of task. For example, we have been able to say that multiple classification, transitivity and serial order tasks are all level 2, and therefore none of them is mastered by average children below age 5 years. This prediction holds across these tasks which are superficially rather different from each other. We therefore have at least the beginning of a theoretical basis for specifying the limits to children's cognitive performance. From this basis we can go on to predict other, as yet unobserved, limits. Some of this has been done already, and we have made predictions concerning hypothesis testing and oddity tasks (Halford, 1982) which have not all been tested yet, so they amount to predictions which are made ahead of empirical evidence.

Now that we have a definition of what children can and cannot do as a function of age we can proceed to make predictions about curriculum-relevant tasks. A modest beginning has been made here with the mathematics tasks mentioned earlier. We will therefore illustrate the practical utility

of the theory by spelling out some of its implications for mathematics education.

It implies that preschool children should be able to master mathematical concepts which depend on single binary relations. That is, they should be able to master object size and set size relations, so work on comparisons of objects and sets should be suitable for them. However, until about mental age 5 children should not be able to deal with concepts which are defined by a composition of two binary relations. Therefore, understanding of order and transitivity should not be possible, nor should binary operations such as addition and subtraction be understood. Care is required in applying predictions of this type. Notice that I am talking about the concepts which children can represent symbolically or mentally. Nothing in what I have said precludes children playing successfully with blocks or other objects which need to be ordered with respect to size. It is quite possible to perform such tasks by strategies which do not require representation of the order or transitivity concepts--e.g., by trial-and-error, or by considering one relation at a time.

I am saying however that most preschool children will not understand quantification above the level of a nominal scale. The reason is that such quantification entails the concept of order, and preschool children cannot, if I am right, understand serial order. Furthermore, they will not be able to mentally represent or understand binary operations such as arithmetical addition and multiplications.

Between the ages of 5 and 10 years children should be able to acquire an understanding of order, transitivity, and arithmetical operations, and should acquire basic quantitative concepts up to the level of an interval scale. However, algebraic manipulations, which involve rules relating to compositions of operations would not be understood.

The levels which I have outlined are not like Piagetian stages for several reasons. Firstly, I do not say that a child belongs to a particular stage. Thus nowhere has it been said that a child or group of children belongs to any specific level. It is rather that working memory provides a shifting upper limit to the concepts which can be attained. Children will not acquire all the concepts at a particular level similtaneously, nor will their acquisition necessarily be sudden. Development of adequate working memory capacity for a particular level merely qualifies the child to profit from appropriate experience with that level of task. Once the child is capable of learning concepts at a particular level, the particular concepts acquired will depend on the specific experiences provided. Thus this paper aims to define the limits to what can be learned, but says nothing about the learning process per se, a topic which is covered by Professor Wallace's paper.

Finally, I am not trying to be pessimistic about children's capabilities. Some quite complex concepts belong to levels 1 and 2, and virtually all the concepts which we adults use can be expressed as level 3 systems. Simple examples have been used for illustrative purposes, but it is possible to construct quite complex level 1 tasks, thus the limits do not necessarily mean that all the concepts used by young children must be simple. What they do mean is that there are limits to the amount of information which can be synthesised in a single decision or a single representation. This is a very real limitation as we have seen but, as we have also seen, it does not prevent children from doing some quite clever things.

## Reference Notes

1. Halford, G. S. Can young children integrate premises in transitivity and serial order tasks? Paper submitted.

2. Halford, G. S., Bain, J. D., & Maybery, M. T. Does concurrent memory load intefere with reasoning? Paper submitted.

3. Halford, G. S., & Kelly, M. E. On the basis of early transitivity judgments. Paper submitted.

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Footnote

[1]This chapter was presented as a paper to the 20th Congress of the International Association of Applied Psychology, University of Edinburgh, July 25-30, 1982.

Table 1

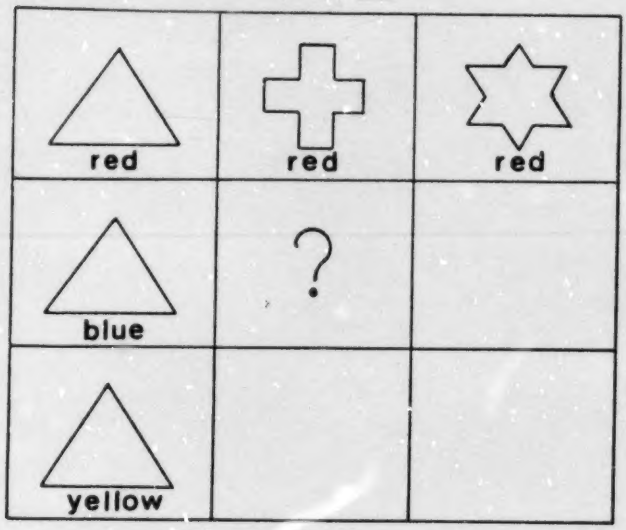
Sample of mathematical problems used by Halford (1978) together with proportions correct for grades 4 to 7

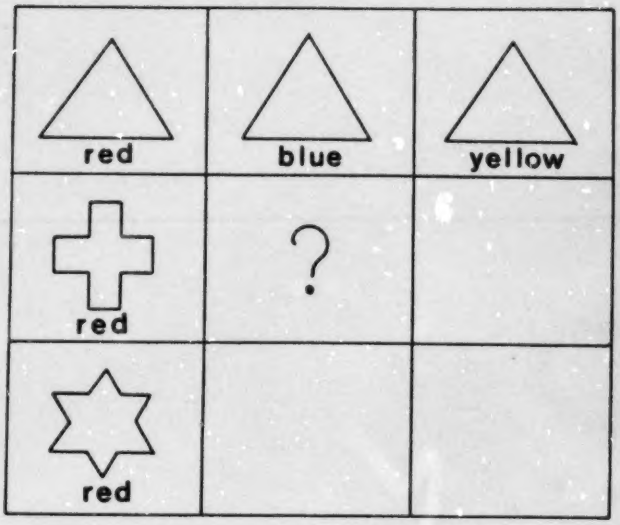
| | Item type | Proportion of correct responses | | | |
|---|---|---|---|---|---|
| | | G4 | G5 | G6 | G7 |
| Level 2 | | | | | |
| | (3 x 5) o 5 = 10 | .65 | .79 | .75 | .94 |
| Level 3 | | | | | |
| | (4 o 2) * 3 = 2 | .19 | .45 | .55 | .64 |

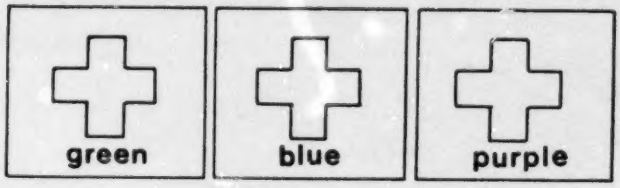
## Figure Caption

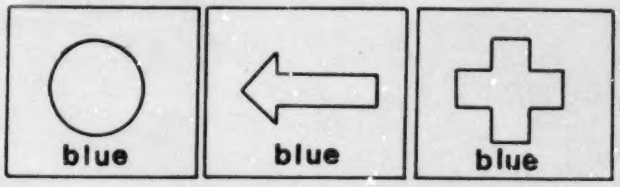
Figure 1. Matrix completion task used to manipulate the amount of information which must be considered to select an item for each square.

LEVEL 1




LEVEL 2